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A world-volume perspective on the recombination of intersecting branes

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Abstract: We study brane recombination for supersymmetric configurations of intersecting branes in terms of the world-volume field theory. This field theory contains an impurity, corresponding to the degrees of freedom localized at the intersection. The Higgs branch, on which the impurity fields condense, consists of vacua for which the intersection is deformed into a smooth calibrated manifold. We show this explicitly using a superspace formalism for which the calibration equations arise naturally from F- and D-flatness.

Keywords: D-branes, Brane Dynamics in Gauge Theories, Tachyon Condensation, Supersymmetric Effective Theories.

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Contents

1. Introduction

It is well known that a supersymmetry preserving set of intersecting branes may merge to form a single brane on a smooth calibrated surface (see [1] for a review). This is expected to occur via the condensation of world-volume degrees of freedom localized at the intersection. However, the manner in which the Higgs branch of the world-volume gauge theory describes smooth calibrated surfaces has not been studied explicity in much detail. Filling this gap is the aim of this note. We shall do so using a superspace for which calibrated geometries on the Higgs branch arise naturally as solutions of F- and D-flatness conditions. The gauge theories which we write down are suitable for describing both the Coulomb branch on which the branes are separated and the Higgs branch on which the branes have merged on a calibrated surface.

These gauge theories contain impurities, given by degrees of freedom constrained to the lower-dimensional subspace where the branes intersection It is convenient to describe such a field theory by a superspace which spans the space-time directions of the impurity. This superspace may also be used to described the fields in the ambient space which are not localized at the impurity. The formalism in which degrees of freedom in higher dimensions are described in a lower-dimensional superspace was developed originally in [2, 3] and has been applied to study a number of theories, including the defect conformal theories which arise for systems of interesecting branes [4]–[7].

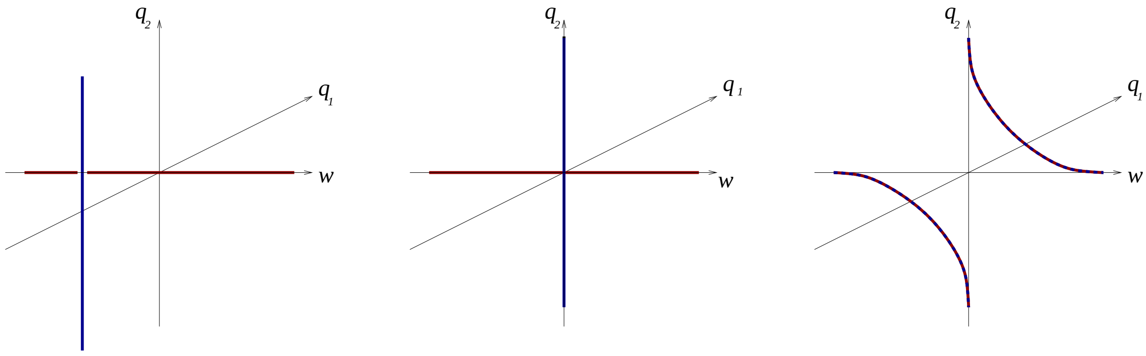

Figure 1: Branes touching and the intersection being resolved.

In the context of brane intersections preserving eight supercharges, the holomorphic curves which arise on the Higgs branch are solutions of the F- and D-flatness conditions for a superspace which spans the mutual coordinates of the intersection. For triple (and quadruple) intersections preserving four supercharges, we will find F- and D-flatness corresponding to special lagrangian conditions. These conditions will take the form of the equations of motion of an abelian Chern-Simons theory and a gauge fixing respectively.

For a particular triple intersection of D6-branes preserving four supercharges, the Higgs branch solutions of the F- and D-flatness conditions correspond to the union of a special lagrangian plane and a holomorphic curve times a line. This configuration lifts to a G_2 manifold in M-theory which shares many features with G2-manifolds discussed in [8, 9].

There are a number of reasons to be interested in the process of brane recombination. In the context of intersecting brane-worlds [10]–[13], see also [14, 15, 16], the Higgs mechanism is believed to be realized by brane-recombination [17]. In this setting intersecting branes merge via open string tachyon condensation. The supersymmetric triple D6-brane intersection which we will study provides a controlled toy model with which to explicitly demonstrate the brane-world Higgs mechanism. A precise treatment of recombination by tachyon condensation has not been given and would seem to require string field theory, except in certain small angle approximation [18]. Approximate treatments have been given using effective tachyon field theories [18]–[21]. For the supersymmetric brane recombination which we will describe, the field theory treatment suffices. Recombination in a supersymmetric case was also considered in [20]. Another reason to be interested in brane recombination is that it is closely related (via U-dualities) to processes such as the brane collision in ekpyrotic [22] or cyclic universes [23]. In this context it is useful to have a description which allows to study the dynamics of transitions between the Coulomb branch, with separated branes, and the Higgs branch on which the branes have recombined.

The organization of this paper is as follows. In section 2, we discuss intersecting pairs of branes which preserve eight supercharges and wrap holomorphic curves on the Higgs branch. This section reviews and expands upon results in [5]. In section 3, we consider intersections preserving four supercharges, for which the F and D-flatness conditions are the the special lagrangian conditions. We study the particular example of three (and four) intersecting D6-branes whose geometry is discussed in section 4. Finally, in section 5, we numerically compute correlations of fields on the different asymptotic regions of branes which have recombined into a holomorphic curve $xy = c$. We find that the correlation vanishes in the singular limit $c \to 0$ in which the branes can separate.

2. Holomorphic curves from intersecting branes preserving eight supercharges

The low energy dynamics of intersecting branes is described by a field theory with impurities. Some of the earliest studies of such impurity field theories may be found in [24, 25, 26]. In many instances, this theory is a non-trivial defect conformal field theory [27, 28, 29, 4, 5, 6, 7]), or dCFT. In this section we review the superspace description of defect field theories and expand upon some results [5] concerning the Higgs branch of the dCFT describing intersecting D3-branes. Although the world-volume of this dCFT is the singular space $xy = 0$, we will explicitly see that the classical Higgs branch has a geometric interpretation as the smooth resolution $xy = c$.

Consider two stacks of D3 branes, one of which spans the directions $x^{0,1,2,3}$, while the other spans $x^{0,1,4,5}$. The two stacks are at the origin in the transverse $x^{6,7,8,9}$ directions. In addition to the $\mathcal{N} = 4$ gauge theory that lives on each stack of parallel D3-branes, there are additional massless fields that arise from open strings stretching between the orthogonal branes. These fields are localized at the 1+1 dimensional intersection. There is an unbroken $(4, 4)$ supersymmetry which includes translations in the $0, 1$ directions. The $(4, 4)$ algebra is a common subalgebra of the two four-dimensional $\mathcal{N} = 4$ algebras associated with each parallel stack of D3-branes.

Although the theory contains both two- and four-dimensional degrees of freedom, one can write the action using a two-dimensional superspace. The degrees of freedom which propagate in four dimensions are described by superfields with continuous indices which parameterize the world-volume directions transverse to the intersection. The directions parallel to the intersection are included in the superspace. In [5] the action for this theory was constructed using two-dimensional $(2, 2)$ superspace. The formalism which we will use below is trivially generalized to intersecting D-branes of other dimensions, such as D5-branes intersecting over four dimensional $\mathcal{N} = 4$ superspace.

The superfields on the first D3-brane are functions of $(x^0, x^1, \theta, \bar{\theta}; w, \bar{w})$. The superspace is spanned by $(x^0, x^1, \theta, \bar{\theta})$, while $w = x^2 + ix^3$ should be thought of as a continuous index. The necessary $(2, 2)$ superfields are a vector superfield V and three chiral superfields Φ, Q_1, and Q_2. While this resembles the four-dimensional $\mathcal{N} = 1$ superfield content of the $\mathcal{N} = 4$ theory, the component fields are distributed very differently. The gauge connections $A_{0,1}$ are contained in V, while $A_2 + iA_3$ is the lowest component of a chiral superfield Φ. This chiral superfield transforms inhomogeneously under $\mathrm{U}(N)$ gauge transformations with non-trivial dependence on the index w. Gauge transformations are parameterized by families of chiral superfields Λ labelled by $w, \bar{w}$. Two of the six adjoint scalars are contained in the $\theta\bar{\theta}$ components of V or, equivalently, the lowest component of a twisted chiral superfield, which is $\Sigma = \frac{1}{2}\{e^{gV}\bar{D}_+ e^{-gV}, e^{-gV} D_- e^{gV}\}$. The four remaining adjoint scalars comprise the lowest components of the chiral superfields Q_1 and Q_2. In $(2,2)$ superspace, the four-dimensional $\mathcal{N}=4$ action is

$$S_{D3} = \frac{1}{g^2}\int d^2x\, d^2w\, d^4\theta\, \mathrm{tr}\left(\Sigma^\dagger\Sigma + (\partial_w + g\bar{\Phi})e^{gV}(\partial_{\bar{w}} + g\Phi)e^{-gV} + \sum_{i=1,2} e^{-gV}\bar{Q}_i e^{gV} Q_i\right) + \\ + \int d^2x\, d^2w\, d^2\theta\, \mathrm{tr}\left(Q_1[\partial_{\bar{w}} + g\Phi, Q_2]\right) + \mathrm{c.c.}\,, \tag{2.1}$$

We shall take (2.1) as the action for degrees of freedom propagating on the first stack of D3-branes. The superfields on the second stack of D3-branes are functions of $(x^0, x^1, \theta, \bar{\theta};\, y, \bar{y})$ where $y = x^4 + ix^5$. Their contribution to the action is just like (2.1) with $N \to N'$, $w \to y$ and primes added to all superfields.

The remaining degrees of freedom are strictly two-dimensional and arise from strings stretched between the orthogonal stacks of D3-branes. These are described by two chiral superfields B and $\tilde{B}$ in the $(N, \bar{N}')$ and $(\bar{N}, N')$ representations of $\mathrm{U}(N)\times\mathrm{U}(N')$. Together these form a $(4,4)$ hypermultiplet. The part of the action containing these fields is

$$S_{D3-D3'} = \int d^2x d^4\theta\, \mathrm{tr}\left(e^{-gV'}\bar{B}e^{gV}B + e^{-gV}\bar{\tilde{B}}e^{gV'}\tilde{B}\right) + \frac{ig}{2}\int d^2x\, d^2\theta\, \mathrm{tr}\left(B\tilde{B}Q_1 - \tilde{B}BQ_1'\right). \tag{2.2}$$

From now on, we will not write out the explicit dependence on the coupling constant g anymore, which is easily reintroduced as it always enters as a prefactor of the V and Φ superfields.

The lowest components of Q^1 and Σ, as well as their primed counterparts, correspond to adjoint scalars describing fluctuations in the directions $x^{6,7,8,9}$ transverse to both stacks of D3-branes. This can be seen by noting that expectations values for these fields give mass to the defect fields B and $\tilde{B}$. The lowest component of Q_2 describes fluctuations of the first stack of D3-branes in the $x^{4,5}$ directions which are tangential to the other stack of D3-branes (the D3$'$ branes). The lowest component of Q_2' describes fluctuations of the the D3$'$-branes in the directions $x^{2,3}$ tangential to the D3-branes.

2.1 Supersymmetric solutions: branches of the moduli space

To determine the supersymmetric vacua, we look for the gauge equivalence classes of solutions to the D- and F-flatness equations. Earlier investigations of moduli spaces of theories with impurities appeared in [26, 30]. The vanishing of the F-terms in this theory requires:

$$F_{Q_1} = \partial_{\bar{w}} q_2 + [\phi, q_2] + \delta^{(2)}(w) b\tilde{b} = 0\,, \tag{2.3}$$
$$F_{Q_2} = \partial_{\bar{w}} q_1 + [\phi, q_1] = 0\,, \tag{2.4}$$
$$F_{\Phi} = [q_1, q_2]\,, \tag{2.5}$$
$$F_{Q_1'} = \partial_{\bar{y}} q_2' + [\phi', q_2'] + \delta^{(2)}(y)\tilde{b}b = 0\,, \tag{2.6}$$
$$F_{Q_2'} = \partial_{\bar{y}} q_1' + [\phi', q_1'] = 0\,, \tag{2.7}$$
$$F_{\Phi'} = [q_1', q_2']\,, \tag{2.8}$$
$$F_B = \tilde{b}q_1\delta^{(2)}(w) - q_1'\tilde{b}\delta^{(2)}(y) = 0\,, \tag{2.9}$$
$$F_{\tilde{B}} = q_1 b\delta^{(2)}(w) - bq_1'\delta^{(2)}(y) = 0\,. \tag{2.10}$$

Throughout this article, we write the scalars which are the lowest components of a chiral superfield in lower-case. The vanishing of the D-terms requires

$$D = \partial_w \phi - \partial_{\bar{w}} \phi^\dagger + [\phi, \phi^\dagger] + [\sigma, \sigma^\dagger] + [q_1, q_1^\dagger] + [q_2, q_2^\dagger] + \delta^{(2)}(w)(bb^\dagger - \tilde{b}^\dagger \tilde{b}) = 0 \,, \qquad (2.11)$$

and there is a similar equation for the second stack with all fields primed and w replaced by y.

In the $\mathcal{N} = (2,2)$ superspace used here, not all terms in the scalar potential come from D- and F-terms. This is due to the fact that also the vector superfield V contains the scalar σ in its $\theta\bar{\theta}$ component. Thus, in the expansion of the kinetic terms of the chiral superfields, there are additional terms of the form

$$(\partial_w \bar{\sigma} + [\bar{\phi}, \bar{\sigma}])(\partial_{\bar{w}} \sigma + [\phi, \sigma]) \,, \qquad (2.12)$$

as well as similar quartic terms involving q_1, q_2 and b arising from the Kähler potentials in (2.1) and (2.2). Together with the square of F- and D-terms, these terms finally yield a potential that is invariant under the $O(4)$ R-symmetry under which q_1 and σ together transform in the vector representation.[1]

In looking for solutions of the F- and D-term equations, we shall take all gauge fields to vanish. First consider the D-term equation (2.11). Assuming that the q-fields are regular the coefficient of the δ-function has to vanish. Thus we find

$$\tilde{b}\tilde{b}^\dagger = b^\dagger b \,, \qquad (2.13)$$

a vev for b always implies a similar vev for $\tilde{b}$.

We can simultaneously diagonalize q_1 and q_1' at $w = 0$ and $y = 0$ respectively since they are acted upon by different gauge groups. (2.10) then becomes

$$0 = b_{i'j} q_{1jj}(0) - q'_{1i'i'}(0) b_{i'j} = b_{i'j}(q_{1jj}(0) - q'_{1i'i'}(0)) \,, \qquad (2.14)$$

where the indices i, j and i', j' denote $\mathrm{SU}(N)$ and $\mathrm{SU}(N')$ indices, respectively. (2.14) is satisfied if $b_{i'j}$ or $q_{1jj}(0) - q'_{1i'i'}(0)$ vanish. The former corresponds to the Coulomb branch of the gauge theory where the orthogonal branes are seperated, while the latter corresponds to the Higgs branch. A similar analysis holds for $\tilde{b}$ instead of b. The fields b and $\tilde{b}$ may only condense on the Higgs branch, where they are massless.

One might worry that the q field in the (2.14) is always evaluated at $w = 0$ as required by the δ-function in (2.9) and not at $w = q_2'$. Van Raamsdonk [31] has investigated this question from a T-dual perspective. He found that in order to remedy this concern, (2.9) should contain $\exp(\alpha' q_1' \partial_w)$ factors. However this exponential is higher order in α' and can be neglected to the order we are working at.

Equations (2.5) and (2.8) show that we can simultaneously diagnolize q_1, q_2 and q_1', q_2'. There is no 'non-commutative' or 'fuzzy' geometry associated with this system and we can treat all adjoint fields as effectively abelian. The brane recombination always occurs pairwise. This means for example that one can not simultaneously have non-zero $b_{11'}$ and

[1] We thank the referee for stressing this point.

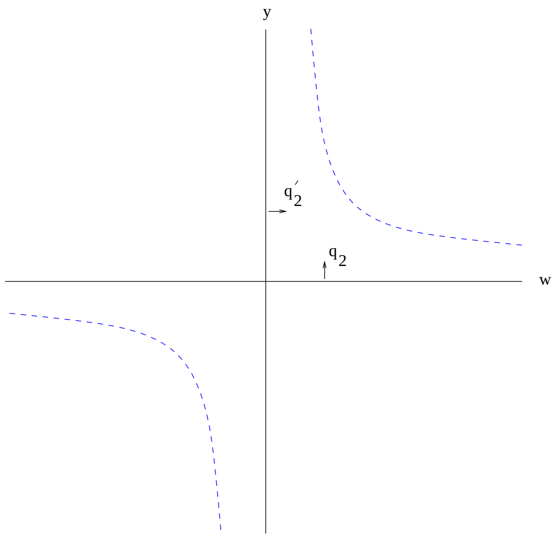

Figure 2: Resolution of intersecting branes $wy = 0$ to $wy = c$ on the Higgs branch.

$b_{12'}$, which would recombine three branes. Pairwise recombination is required because of the D-flatness constraint $b^\dagger b = \tilde{b}\tilde{b}^\dagger$ and because equations (2.3) and (2.6) imply that $b\tilde{b}$ and $\tilde{b}b$ are diagonal.[2] Without loss of generality, we will from now on consider the abelian case, in which one D3-brane intersects one D3$'$ brane.

Equation (2.4) implies that q_1 is a holomorphic function of w, a condition on the embedding coordinates that is well known to be necessary for a supersymmetric brane configuration, see for example [32, 33]. The condition (2.3) implies that $q_2(w)$ is holomorphic except at the origin $w = 0$.

The solution of (2.3) is

$$q_2(w) = \frac{b\tilde{b}}{2\pi i w} + h(w)\,, \tag{2.15}$$

where $h(w)$ is a holomorphic function of w. It is easiest to study brane recombination in the case in which $h(w)$ vanishes, corresponding to boundary conditions $q_2(w) \to 0$ at $w \to \infty$. With boundary conditions at infinity corresponding to the original configuration of orthogonal intersecting branes, the unique solution of (2.3) and (2.6) is

$$q_2(w) = \frac{\tilde{b}b}{2\pi i w}\,, \qquad q_2'(y) = \frac{b\tilde{b}}{2\pi i y}\,. \tag{2.16}$$

Recall that q_2 (q_2') describes the transverse fluctuations of the D3-brane (D3$'$-brane) in the directions $y(w)$ tangential to the D3$'$-brane (D3-brane). The geometry of the D3-branes is obtained by making the replacements $q_2 \to \alpha' y$ and $q_2' \to \alpha' w$ in (2.16), which can then be collectively written as

$$wy = \frac{1}{2\pi i} b\tilde{b}\alpha'. \tag{2.17}$$

[2]This is different from the T-dual system of D0-branes resolving as instantons in D4 branes: There, each instanton sits in a SU(2) subgroup and thus involves two D4 branes. It might however be that our system corresponds to the limit of infinte separation of branes of dyonic instantons that becomes regular after two T-dualities.

In other words the original pair of orthogonal branes now lie on the same holomorphic curve. Furthermore these branes have merged such that there is actually only one D3-brane on the holomorphic curve. This follows from the fact that the gauge group $\mathrm{U}(1) \times \mathrm{U}(1)$ is broken to the diagonal $\mathrm{U}(1)$ when $b\tilde{b} \neq 0$.

There are of course a much broader class of supersymmetric intersections and holomorphic curves which arise from brane recombination on the Higgs branch. This includes the the recombination of branes at angles, for which $h(y)$ is linear function of y [34].

3. Special lagrangian three-folds and multiple brane intersections

In the previous section, we discussed intersecting D-branes preserving eight supersymmetries merging into a single holomorphic curve on the Higgs branch. We shall now consider intersecting configurations which preserve four supercharges and merge into a special lagrangian three-fold on the Higgs branch. Specifically, we consider D6-branes spanning four common Minkowski space directions, while the remaining three directions are embedded in a Calabi-Yau three-fold.

A special lagrangian three-fold (see [35] for a review) is a real three-dimensional surface embedded in a Calabi-Yau three-fold such that

$$\omega|_L = \mathrm{Im}\,\Omega|_L = 0 \tag{3.1}$$

where $\omega|_L$ is the restriction of the Kähler form to the surface L, while $\Omega|_L$ is the restriction of the holomorphic three-form to L. Such a manifold is calibrated with respect to $\mathrm{Re}(\Omega)$ and is volume minimizing in its homology class. The role of the special lagrangian conditions as a BPS condition was first discussed in [36], (see also [37] and references therein). However, the recombination of intersecting branes into smooth special lagrangian manifolds has not been explicitly discussed from the point of view of the world-volume theory.

To study the geometries which arise on the Higgs branch of the world-volume gauge theory of the intersecting D6-branes, it is most convenient to use a $\mathcal{N}=1$ superspace in four dimensions, corresponding to the dimensions spanned by the intersection. Note that our discussion is readily generalized to intersecting branes of other dimension by T-duality. For intersecting D3-branes, our discussion goes through almost identically after replacing $\mathcal{N}=1, d=4$ superspace with $\mathcal{N}=2, d=1$ superspace (which is also associated with four supercharges). We will begin by studying the action of a stack of parallel D6-branes in a four-dimensional $\mathcal{N}=1$ superspace.[3] It is then easy to construct the action for intersecting configurations with four common directions and four unbroken supersymmetries.

3.1 7-dimensional maximally supersymmetric SYM in $\mathcal{N}=1$, $D=4$ superspace

The four dimensional $\mathcal{N}=1$ superspace representation of the D6-brane action was constructed in [3] and is related to a construction discussed earlier in [2]. The four-dimensional $\mathcal{N}=1$ superfields entering the action have the general form $F(x^\mu,\theta,\bar{\theta}|\vec{y})$, where $(x^\mu,\theta,\bar{\theta})$

[3] neglecting couplings to gravity/bulk degrees of freedom.

spans the four-dimensional superspace, and $\vec{y} \sim (y^1, y^2, y^3)$ can be regarded as continuous indices. The necessary degrees of freedom are contained in three chiral fields Φ_I and a vector field V. The action is

$$S = \frac{1}{g^2}\int d^3y\, d^4x\, d^2\theta\, \mathrm{tr}\left[W_\alpha W^\alpha + \epsilon_{ijk}(\Phi_i \frac{\partial}{\partial y^j}\Phi_k + \frac{2}{3} i \Phi_i \Phi_j \Phi_k)\right]\bigg|_{\theta\theta} + \text{c.c.} + \tag{3.2}$$

$$+\frac{1}{g^2}\int d^3y\, d^4x\, d^4\theta\, \mathrm{tr}\, \bar{\Omega}_i e^V \Omega_i e^{-V}\bigg|_{\theta\theta\bar{\theta}\bar{\theta}}, \tag{3.3}$$

where the indices i, j, k take values from 1 to 3, and the superfield

$$\Omega_i = \Phi_i + e^{-V}(i\partial_i - \bar{\Phi}_i)e^V . \tag{3.4}$$

The bosonic fields of the theory consist of seven gauge connections $A_{0,1,2,3,4,5,6}$ and three hermitean adjoint scalars $X_{7,8,9}$ describing transverse fluctuations of the D6-brane. These are distributed amongst the four-dimensional $\mathcal{N} = 1$ superfields as follows,

$$V \to A_{0,1,2,3}\,, \qquad \Phi_i \to A_{4,5,6}\,, \qquad X^{7,8,9}\,. \tag{3.5}$$

The combination $A_{i+3} + iX^{i+6}$ for $i = 1, 2, 3$ is the lowest component of the chiral superfield Φ_i. Gauge transformations are chiral superfields $\Lambda(\vec{x}^\mu, \theta; \vec{y})$, which act in the following way,

$$e^V \to e^{i\Lambda^\dagger} e^V e^{-i\Lambda}\,, \tag{3.6}$$

$$i\partial_i - \Phi_i \to e^{i\Lambda}(i\partial_i - \Phi_i)e^{-i\Lambda}\,. \tag{3.7}$$

Under these transformations, the superfield Ω_i, transforms as

$$\Omega_i \to e^{i\Lambda}\Omega_i e^{-i\Lambda}\,. \tag{3.8}$$

Seven-dimensional Lorentz invariance is not manifest but becomes apparent upon integrating out auxiliary fields and performing suitable field redefinitions. Note the resemblance of the superpotential to a Chern-Simons action.

The action we have written is for a D6-brane in flat space, in which we have taken the SYM approximation of the Dirac-Born-Infeld action. As we shall see shortly, a consideration of the full Dirac-Born-Infeld action should lead to a modified D-flatness condition but not a modified F-flatness condition. If the D6-brane wraps a three-cycle of a Calabi-Yau, the D-terms will be further modified, although the diffeomorphism invariant Chern-Simons superpotential will not change.

3.2 Special lagrangians from F- and D-flatness

Let us now consider the F- and D-flatness conditions for the action (3.2). Henceforward it is convenient to write X^{i+6} as X^i, A_{i+3} as A_i and $\partial/\partial y^i = \partial_i$ where $i = 1, 2, 3$. F-flatness gives $\partial W/\partial\phi_i = 0$, or

$$D_i X_j - D_j X_i = 0\,, \tag{3.9}$$

$$F_{ij} - [X_i, X_j] = 0\,, \tag{3.10}$$

where F_{ij} is the gauge field strength. The D-flatness condition is

$$D_i X_i = 0 \,. \tag{3.11}$$

We consider only abelian solutions with $F_{ij} = 0$, in which case F- and D-flatness become

$$\partial_i X_j - \partial_j X_i = 0 \,, \tag{3.12}$$

$$\partial_i X_i = 0 \,. \tag{3.13}$$

The solutions of (3.12) and (3.13) determine the embedding of the D6-brane in $\mathbb{C}^3$, which we shall parameterize by the complex coordinates $u^i \equiv y^i + i\alpha' X_i$. It is now easy to show that equations (3.12) and (3.13) are linearized special lagrangian conditions.[4] This can be seen as follows. For $\mathbb{C}^3$, the Kähler form is $\omega = du^i \wedge d\bar{u}^i$, while the holomorphic three-form is $\Omega = du^1 \wedge du^2 \wedge du^3$. Restricted to the embedding $X_i(y^j)$, the differentials du^i satisfy

$$du^i = dy^i + i\alpha' \frac{\partial X_i}{\partial y^j} dy^j \,. \tag{3.14}$$

So that the condition for a lagrangian manifold is just

$$\omega|_L = du^i \wedge d\bar{u}^i = 2i\alpha' dy^i \wedge dy^j \, \partial_i X_j = 0 \,, \tag{3.15}$$

which is equivalent to F-flatness (3.12). A special lagrangian manifold satisfies the additional condition

$$\mathrm{Im}\,\Omega|_L = \mathrm{Im}\, du^1 \wedge du^1 \wedge du^3 = dy^i \wedge dy^j \wedge dy^k \, \alpha' \left(\partial_i X_i + \alpha'^2 \det(\partial_i X_j) \right) = 0 \,. \tag{3.16}$$

The determinant in (3.16) is with respect to the matrix indices ij. If we use (3.12), to write $\vec{X}$ as a gradient of a scalar potential f, then we recognize in 3.16 the three dimensional version of the special lagrangian condition $0 = \det(Id + \mathrm{Hess}(f))$ that was discussed in [35]. Up to the determinant term, (3.16) agrees with the D-flatness condition (3.13). We assume that the determinant arises from the D-flatness condition for the full Dirac-Born-Infeld lagrangian. Unfortunately, the supersymmetric versions of Dirac-Born-Infeld actions are quite involved and the known superspace descriptions [39] do not include scalar fields.

In the special case in which $\det(\partial_i X_j)$ vanishes, the solutions of the F- and D-flatness equations are shared by the SYM and DBI actions. This is similar to the case of the BIon for which certain solutions are shared among Yang-Mills and Born-Infeld theories as the non-linear terms of the Born-Infeld field equation vanish for them [40, 41]. We find such a situation for the Higgs branch of a triple D6-brane intersection. The special lagrangians which arise in this case are the product of a holomorphic curve with a line, for which the determinant vanishes.

Note that if one were to view X_i as a gauge field, (3.15) and (3.16) would require this gauge field to be a flat connection in a particular non-linear gauge. This is a very special gauge condition in the following sense. The SU(3) invariance of the conditions (3.15) and (3.16) means that one can exchange coordinates with "gauge" fields such that connections remain flat and the form of the gauge condition is unchanged.

[4]SLAG conditions were also discussed in terms of graphs of functions in [35] and in [38], although there in a different formalism.

3.3 Knots, non-compact three-cycles, and resolved SLAG intersections

The F- and D-flatness equations,

$$\partial_i X_j - \partial_j X_i = 0\,, \tag{3.17}$$
$$\partial_i X_i + \alpha'^2 \det(\partial_i X_j) = 0\,, \tag{3.18}$$

have solutions which can be interpreted as flat connections subject to a particular gauge condition. From the example of the double intersection discussed in section 2, we expect that it is also necessary to include delta function sources in these equations when considering the recombination of intersecting branes. In particular, the flat connection condition is overly restrictive, as can be seen in the following example.

Consider a special lagrangian three-cycle $\mathcal{C} \times \mathbb{R}$ where $\mathcal{C}$ is the holomorphic curve $w = c/v$, with c real and

$$w \equiv X_2 + iX_1\,, \qquad v = y^1 + iy^2 \qquad X_3 = 0\,. \tag{3.19}$$

Since $\partial_{\bar v} w = 2\pi i c \delta^2(v)$, it follows that

$$\partial_1 X_2 - \partial_2 X_1 = 2\pi c\, \delta^2(y^1, y^2)\,, \qquad \partial_2 X_3 - \partial_3 X_2 = \partial_1 X_3 - \partial_3 X_1 = 0\,, \tag{3.20}$$
$$\partial_1 X_1 + \partial_2 X_2 = 0\,. \tag{3.21}$$

From the point of view of the D6-brane world-volume theory, the first set of conditions (3.20) arises from a modification of the (abelian) Chern-Simons superpotential which includes a holomorphic Wilson line:

$$W = \int_{\Sigma_3} \phi d\phi + i2\pi c \int_{y^1=y^2=0} dy^3 \phi_3\,, \tag{3.22}$$

while the D-term is unchanged. In the limit $c \to 0$ one obtains an intersection of complex planes $wv = 0$ times a line.

One can also include a more general holomorphic Wilson line in the superpotential

$$W = \int_{\Sigma_3} \phi d\phi + ic2\pi \int_{\Sigma_1} \phi\,, \tag{3.23}$$

where Σ_1 is an arbitrary closed or infinite path $\vec{y}^i(s)$. The real part of the F-term is not modified by the Wilson line, such that F-flatness still requires a flat gauge connection. However the imaginary part of the F-term is corrected by a term with delta function support on Σ_1;

$$\mathrm{Im}\, F_{\Phi_k} = \epsilon_{ijk}(\partial_i X_j - \partial_j X_i) - J_k = 0\,, \tag{3.24}$$

where

$$J_k = \int ds \delta^3(\vec{y} - \vec{y}(s)) \frac{dy^k(s)}{ds}\,. \tag{3.25}$$

The solutions of (3.24) belong to a class of non-compact lagrangian manifolds[5] associated with the path Σ_1.

[5]Despite the delta function the lagrangian condition $\omega|_{\mathcal{L}} = 0$ holds at any finite point on the (non-compact) curve which is a solution of the F-flatness equation.

In the context of the deformed conifold $T^*(S^3)$, the existence of a lagrangian manifold passing through every knot in S^3 was pointed out in [42]. With a different motivation in mind, the authors of [42] considered topological open strings in the background of a D6-brane wrapping $\Sigma_3 = S^3$ and a D6-brane on a non-compact lagrangian manifold intersecting S^3 over a knot Σ_1. The solutions of (3.24) in this setting correspond to a recombination of these D6-branes into a single D6-brane on a smooth lagrangian manifold. For topological strings one does not consider the D-flatness condition.

For non-topological strings, one must also consider D-flatness, which leads to a special lagrangian manifold. Although the D-term only takes the form (3.21) on $\mathbb{C}^3$, we expect it still plays a role analogous to a gauge fixing for a non-trivial Calabi-Yau manifold.

For the case of $\mathbb{C}^3$, solutions of (3.24) and (3.21) are smooth non-compact special lagrangian manifolds which should reduce to an intersection in the limit $c \to 0$. As noted above D-flatness can be regarded as a gauge fixing. There may be interesting non-perturbative effects such as Gribov ambiguities. However a unique perturbative expansion can be found if the path $y^i(s)$ is almost straight over distances of order $\sqrt{\alpha'}$ and suitable boundary conditions are imposed at infinity. The perturbative expansion in α' is obtained by writing

$$\vec{X} = \sum_{n=0} \vec{X}_{(n)} \alpha'^{2n} \tag{3.26}$$

and solving

$$\begin{aligned} \vec{\nabla} \times \vec{X}_{(0)} &= \vec{J}, & \vec{\nabla} \cdot \vec{X}_{(0)} &= 0\,, \\ \vec{\nabla} \times \vec{X}_{(n>0)} &= 0\,, & \vec{\nabla} \cdot \vec{X}_{(n>0)} &= \frac{1}{6} \epsilon_{ijk} \epsilon_{i'j'k'} \sum_{\substack{0 \le l,m,p \le n \\ l+m+p=n-1}} \partial^i X^{i'}_{(l)} \partial^j X^{j'}_{(m)} \partial^k X^{k'}_{(p)} \end{aligned} \tag{3.27}$$

The physical origin of the Wilson line in the effective superpotential (3.23) is the condensation of degrees of freedom localized at the intersection of the brane we have been discussing and another brane which meets it over the common superspace directions and the path $\Sigma_1 : y(s)$. Since the coordinate s is transverse to the superspace, kinetic terms in this direction should arise from a superpotential. For the degrees of freedom at the intersection, this superpotential takes the form

$$W_{\text{impurity}} = \int ds B(s) \left(\partial_s - i \left(\Phi_i(\vec{y}(s)) - \Phi'_i(\vec{y}(s)) \right) \frac{dx^i}{ds} \right) \tilde{B}(s)\,, \tag{3.28}$$

analogous to (2.2). Here $B(s)$ and $\tilde{B}(s)$ are an infinite class of chiral superfields labelled by s which describe the degrees of freedom at the intersection. B is in the bifundamental representation of the gauge group on the two D6-branes, while $\tilde{B}$ is in the conjugate representation. The chiral superfield Φ'_i is the counterpart of Φ_i on the second D6-brane. A similar expression also appeared in [42]. Note that integrating out F-terms for B and $\tilde{B}$ gives bosonic kinetic terms in the s direction. The effective superpotential (3.23) arises if the impurity fields B and $\tilde{B}$ condense, i.e. if $\langle b\tilde{b} \rangle = c$ where $b, \tilde{b}$ are the lowest components of $B, \tilde{B}$. F-flatness now gives

$$F_{\Phi_i} = 0 = \epsilon_{ijk}(\partial_j \phi_k - \partial_k \phi_j) - \int ds \delta^3(\vec{y} - \vec{y}(s)) \frac{dy^i(s)}{ds} \tilde{b}(s) b(s)\,. \tag{3.29}$$

Note that $\partial_i F_{\Phi_i} = 0$ implies $\partial_s b(s)\tilde{b}(s) = 0$. Recalling that $\phi_i = A_i + iX_i$, solutions of (3.29) with vanishing gauge fields A_i require $b\tilde{b}$ to be real.

There is also a Kähler potential for the degrees of freedom at the intersection, which, at least in the $\alpha' \to 0$ limit, is of the form

$$\int ds \sqrt{g_{ss}} (\bar{B}(s) e^{V(\vec{y}(s))} B(s) e^{-V'(\vec{y}(s))} + \bar{\tilde{B}}(s) e^{-V(\vec{y}(s))} \tilde{B}(s) e^{V'(\vec{y}(s))}) \,. \tag{3.30}$$

The D-flatness condition is modified by the Kähler term to become

$$i\partial_i X_i + \int ds \sqrt{g_{ss}} \delta^3(\vec{y} - \vec{y}(s))(b^* b - \tilde{b}^* \tilde{b}) = 0 \,. \tag{3.31}$$

The new condition arising from this is the real part $b^* b - \tilde{b}^* \tilde{b} = 0$, which we assume is unaltered when considering the full Dirac-Born-Infeld action.

The vanishing of the F-terms for $\tilde{B}$ and B requires

$$F_{\tilde{B}} = \left(\partial_s - i\left(\phi_i(\vec{y}(s)) - \phi'_i(\vec{y}(s))\right)\frac{dy^i(s)}{ds}\right)\tilde{b} = \left(\partial_s + i\left(\phi_i(\vec{y}(s)) - \phi'_i(\vec{y}(s))\right)\frac{dy^i(s)}{ds}\right)b = 0 \,. \tag{3.32}$$

A similar equation applies for $\tilde{b}$. Since $b\tilde{b}$ is real and independant of s, and $|b| = |\tilde{b}|$, we can write $b = \sqrt{c}\exp(i\theta(s)), \tilde{b} = \sqrt{c}\exp(-i\theta(s))$. For solutions with vanishing gauge field, the real and imaginary parts of (3.32) imply that $\partial_s(\theta(s)) = 0$ and

$$(X_i(\vec{y}(s)) - X'_i(\vec{y}(s)))\frac{dy^i(s)}{ds}\sqrt{c} = 0 \,. \tag{3.33}$$

This equation distinguishes between the Coulomb branch for which $c = 0$ and the branes can separate, and the Higgs branch for which

$$(X_i(\vec{y}(s)) - X'_i(\vec{y}(s)))\frac{dy^i(s)}{ds} = 0 \,. \tag{3.34}$$

On the Higgs branch, the solutions of the F- and D-flatness conditions correspond to the recombination of the intersecting D6-branes into a smooth special lagrangian manifold. Note that the formalism we have been using generalizes trivially to intersections of branes of other dimensions which also preserve four supercharges. The one-dimensional $\mathcal{N} = 2$ superspace action for D3-branes is just a dimensional reduction of (2.1) in which $d^4x \to dt$.

3.4 Triple and quadruple intersections of branes preserving four supercharges

We now consider a configuration of D6-branes in flat space which intersect over three common spatial directions and preserve four supersymmetries. Writing the D6-brane action in a four-dimensional $\mathcal{N} = 1$ superspace as in (3.2) greatly facilitates the construction of the action for this type of intersection. Once we have constructed the action for the intersection we examine the geometry which arises on the Higgs branch. In this case, the special lagrangians which we obtain are holomorphic curves times a line.

	0	1	2	3	4	5	6	7	8	9
	z_0	z_1	z_2	z_3	y_{14}	y_{24}	y_{34}	y_{23}	y_{31}	y_{12}
1	×	×	×	×	×				×	×
2	×	×	×	×		×		×		×
3	×	×	×	×			×	×	×	
4	×	×	×	×	×	×	×			

Table 1: ?????

The D6-brane orientations which we consider are summarized in table 1. The ×'s indicate a direction in which the D6-brane is extended. If there are D6-branes in three or all four of these orientations, then four supersymmetries are unbroken. We shall focus on the case in which there are D6-branes in the first three orientations, in which case it will still be convenient to use the above notation. The directions $z^{0,1,2,3}$ belong to the four dimensional superspace. It is convenient to label the six coordinates transverse to the superspace by x_{AB} where $A \neq B$, A and B run from 1 to 4, and there is no distinction between y_{AB} and y_{BA}. The Ath stack of branes then extends in the directions y_{AB} for three values of B. For example stack two extends in y_{12}, y_{23} and y_{24} while it is localized in y_{13}, y_{14}, and y_{34}.

Associated to the D6-brane of the A'th orientation is a vector multiplet V_A and three chiral multiplets $\Phi^A{}_B$ where $B \neq A$. The lowest component of $\Phi^A{}_B$ contains the gauge connection in the y_{AB} direction and the scalar that describes the position in direction y_{CD}, where none of the labels A, B, C or D are equal. Note that this is also the one direction in which neither brane A nor brane B are extended.

In this notation, the action for a D6-brane in the A'th orientation is

$$\begin{aligned} S_A = &\frac{1}{g^2}\int d^4z\, d^4\theta \prod_E dy_{AE} \sum_{B\neq A} \mathrm{tr}\left(e^{-V_A}\bar{\Omega}^A{}_B e^{V_A}\Omega^A{}_B\right) + \\ &+\frac{1}{g^2}\int d^4z\, d^2\theta \prod_E dy_{AE} \sum_{BCD} \epsilon_{ABCD}\, \mathrm{tr}\left(\Phi^A{}_B\partial_{AC}\Phi^A{}_D + i\frac{2}{3}\Phi^A{}_B\Phi^A{}_C\Phi^A{}_D\right) + \mathrm{c.c.}\,, \end{aligned} \tag{3.35}$$

where

$$\Omega^A{}_B \equiv \Phi^A{}_B + e^{-V_A}(i\partial_{AB} - \bar{\Phi}^A{}_B)e^{V_A}\,. \tag{3.36}$$

The action is invariant under the gauge transformations

$$\begin{aligned} (i\partial_{AB} - \Phi^A{}_B) &\longrightarrow e^{i\Lambda_A}(i\partial_{AB} - \Phi^A{}_B)e^{-i\Lambda_A}\,, \\ e^{V_A} &\longrightarrow e^{i\Lambda_A^\dagger}e^{V_A}e^{-i\Lambda_A}\,. \end{aligned} \tag{3.37}$$

There are also additional fields that live at the intersection of pairs of branes with different orientations. These degrees of freedom are chiral superfields B_{AB}, where B_{AB} and B_{BA} are not equivalent. Under gauge transformations

$$B_{AB} \to e^{i\Lambda_A}B_{AB}e^{-i\Lambda_B}\,, \qquad B_{BA} \to e^{i\Lambda_B}B_{BA}e^{-i\Lambda_A}\,. \tag{3.38}$$

The fields B_{AB} depend on the parameter y_{AB} in addition to superspace coordinates shared by all the branes. The action contains the terms

$$S_{AB} = \frac{1}{g^2}\int d^4z\, d^4\theta\, dy_{AB}\,\mathrm{tr}\left(e^{-V_B}\bar{B}_{AB}e^{V_A}B_{AB} + e^{-V_A}\bar{B}_{BA}e^{V_B}B_{BA}\right) + \\ +\frac{1}{g^2}\int d^4z\, d^2\theta\, dy_{AB}\,\mathrm{tr}(B_{BA}\partial_{AB}B_{AB} + iB_{BA}\Phi^A{}_B B_{AB} - i\Phi^B{}_A B_{BA}B_{AB}) + \mathrm{c.c.} \tag{3.39}$$

The kinetic terms in the y_{AB} direction come from the the superpotential rather than the Kähler potential.

Note that the term $S_A + S_B + S_{AB}$ is fixed by the requirement that it be the action for a double intersection which preserves eight supercharges. Upon compactifying z^2, z^3 and y_{AB}, one must have the action of the double D3-intersection discussed in section 2 (after integrating out auxiliary fields and performing some field redefinitions). Since we consider D6-branes in three or four of the orientations in table 1 there may also be a term in the superpotential involving the twist fields of more than one pair of branes. This term is not fixed by any symmetry, and must be obtained from a string scattering calculation. Since three or more D6-branes intersect over four dimensions, this term is defined only on the superspace coordinates. For the triple intersection, the general form of such a term is

$$S_{ABC} = \frac{1}{g^2}\sum_n \gamma_n \int d^4z\, d^2\theta\, B^n \tag{3.40}$$

where B^n is shorthand for a gauge invariant product of n twist fields in which all three brane indices are represented. As in the case of D3-branes, the B's are dimensionless.[6] Thus γ_n are numbers which must be determined from a string scattering computation. The simplest possible term in (3.40) is $B_{AB}B_{BC}B_{CA}$ The coefficient γ_3. would most easily read of as the strength of a Yukawa coupling between three twist fields. This calculation has been performed in [43, 44, 45] and γ_3 is implicit in those results. As long as it does not vanish, the precise numerical value of γ_3 is not important for our purposes. For the quadruple action there may also be terms S_{ABCD} in which all four brane-indices are represented. The complete action for the system of intersecting branes is the sum of all these single, double, triple and possibly quadruple intersection actions:

$$S = \sum_A S_A + \sum_{AB} S_{AB} + \sum_{ABC} S_{ABC} + S_{ABCD}\,. \tag{3.41}$$

4. F- and D-flatness for the triple intersection

We now look for the solutions of the F- and D-flatness equations to see what geometrical configurations arises when the twist fields condense. Let us first consider the vanishing of the F-term for Φ^A_B:

$$0 = F_{\Phi^A{}_B} = \sum_{CD}\epsilon^A_{BCD}(\partial_{AC}\phi^A{}_D + i\phi^A{}_C\phi^A{}_D) + i\delta^2(y_{AE})|_{E\neq B}b_{AB}b_{BA}\,. \tag{4.1}$$

[6] The dimension of g is $[g] = -3/2$ as appropriate for the Yang-Mills coupling in seven dimensions.

As before, we are interested only in solutions in which the gauge connections vanish. Thus all the ϕ's are anti-hermitian, and we shall write $\Phi^A_B = iX^A_B$. Considering the hermitean part of (4.1) gives

$$\sum_{CD} \epsilon^A_{BCD}[X^A_C, X^A_D] = \delta^2(y_{AE})\big|_{E\neq B}(b_{AB}b_{BA} - b^\dagger_{BA}b^\dagger_{AB})\,. \tag{4.2}$$

As long as the $X's$ are regular, the left and right hand side of (4.2) must vanish separately. This precludes a non-commutative geometry, and we will henceforward just consider abelian equations. The anti-hermitean part of (4.1) gives

$$\sum_{CD} \epsilon^A_{BCD}\partial_{AC}X^A_D = (b_{AB}b_{BA} + b^\dagger_{BA}b^\dagger_{AB})\delta^2(y_{AE})|_{E\neq B}\,. \tag{4.3}$$

which is a special case of (3.24). Regarding X as a magnetic field, equation (4.3) can be viewed as a magnetostatics equation $\vec{\nabla}\times\vec{B} = \vec{J}$. For the case of the triple intersection, there are two orthogonal lines of current as the index B in (4.3) can take two possible values. Current conservation then requires that $\partial_{AB}(b_{AB}b_{BA})$ vanishes (where there is no sum on any of the indices). We therefore look for solutions for which $b_{AB}b_{BA}$ is constant.

Next consider the vanishing of the D-term, which in the abelian case is:

$$\begin{aligned} 0 &= D_{V_A} \\ &= \sum_{B\neq A}\partial_{AB}({\phi^A}_B - {\bar\phi^A}_B) + \sum_{BCD}\epsilon_{ABCD}\delta(y_{AC})\delta(y_{AD})\Big(b_{AB}b^\dagger_{AB} - b^\dagger_{BA}b_{BA}\Big)\,. \end{aligned} \tag{4.4}$$

Considering the real and imaginary parts of (4.4) separately gives

$$b_{AB}b^\dagger_{AB} - b^\dagger_{BA}b_{BA} = 0 \tag{4.5}$$

and

$$\sum_B \partial_{AB}X^A_B = 0\,, \tag{4.6}$$

which corresponds to (3.18) if $\det_{BC}\partial_{AB}X^A_C = 0$.

Finally consider the F-term for the twist fields:

$$0 = F_{B_{AB}} = \partial_{AB}b_{BA} + i{\phi^A}_B b_{BA} - ib_{BA}{\phi^B}_A + \delta(y_{AB})\frac{\partial W_L}{\partial b_{AB}}\,. \tag{4.7}$$

In light of the previous F- and D-flatness conditions, we can write $b_{AB} = c_{AB}e^{i\theta_{AB}(y_{AB})}$ and $b_{BA} = c_{AB}e^{-i\theta_{AB}(y_{AB})}$. Taking $\phi^A_B = iX^A_B$, (4.7) becomes

$$ic_{AB}\partial_{AB}\theta_{AB}(y_{AB}) - (X^A_B - X^B_A)c_{AB} + e^{-i\theta_{AB}(y_{AB})}\frac{\partial W_L}{\partial b_{AB}}\delta(y_{AB}) = 0\,, \tag{4.8}$$

where W_L is the part of the superpotential localized at the point which where all the branes intersect, $S_{ABC} \sim \int W_L$. Solutions for which X is regular (describes a smooth surface) and $\partial_{AB}\theta_{AB} = 0$ require

$$\frac{\partial W_L}{\partial b_{AB}} = 0\,, \tag{4.9}$$

$$(X^A_B - X^B_A)c_{AB} = 0\,. \tag{4.10}$$

Solutions of equation (4.10) distinguish between the Coulomb branch with $c_{AB} = 0$ and the Higgs branch with $X^A_B - X^B_A = 0$. If one had only the lowest order term, $W_L \sim \gamma_3 B_{AB}B_{BC}B_{CD}$, (4.9) would require b_{AB} to vanish for all but one pair of indices A and B. We have no reason to exclude higher order terms in W_L, however it is easy to see that a vanishing b_{AB} for all but one pair of indices A and B remains a solution of (4.9). This is because W_L must involve all three indices, since terms with just one pair of indices would live in five rather than four dimensions. Actually, gauge invariance requires each index to appear at least twice. New solutions will in general exist in the presence of higher order terms. However, for a canonical normalization of the B fields, such that the two point function is independent of g in the weak coupling limit, the extra solutions approach infinity in field space in the weak coupling limit. We will not consider such solutions here.

When the b_{AB} condense for one pairing of indices, two branes recombine on the Higgs branch while the third is not deformed. The same configuration was also obtained by Lambert [46] using a supersymmetry condition arising from bulk supergravity. For the branes which recombine, (4.6) and (4.3) have the structure $\vec\nabla \times \vec X(\vec y) = c^2\delta^2(y^1,y^2)$ and $\vec\nabla\cdot\vec X(\vec y) = 0$, whose solution[7] is

$$X^3 = 0\,, \qquad X^2 + iX^1 = \frac{c^2}{y^1+iy^2}\,. \tag{4.11}$$

This corresponds to a holomorphic curve Σ constrained to lie in the $X^3 = 0$ plane, times a line parametrized by the coordinate y_3. Furthermore the third D6 brane remains flat. To be more specific, consider the case in which b_{12} and b_{21} condense. We then have the following solution for each of the three orthogonal branes:

$$X^1_2 = 0\,, \qquad X^1_3 + iX^1_4 = \frac{c}{y_{14}+iy_{31}}\,, \tag{4.12}$$

$$X^2_1 = 0\,, \qquad X^2_3 + iX^2_4 = \frac{c}{y_{24}+iy_{23}}\,, \tag{4.13}$$

$$X^3_1 = X^3_2 = X^3_4 = 0\,. \tag{4.14}$$

The X fields are identified with fluctuations in a particular direction as

$$\begin{aligned} &X^1_2 \sim y_{34}\,, \qquad X^1_3 \sim y_{24}\,, \qquad X^1_4 \sim y_{23}\,,\\ &X^2_1 \sim y_{34}\,, \qquad X^2_3 \sim y_{14}\,, \qquad X^2_4 \sim y_{31}\,,\\ &X^3_1 \sim y_{24}\,, \qquad X^3_2 \sim y_{14}\,, \qquad X^3_4 \sim y_{12}\,. \end{aligned} \tag{4.15}$$

Thus (4.12) and (4.13) describe a holomorphic curve times a line, while (4.14) is a special lagrangian plane. Altogether, we may write the special lagrangian manifold L as

$$L = \{y_{34} = 0,\ (y_{24}+iy_{23})(y_{14}+iy_{31}) = c\} \cup \{y_{14} = y_{24} = y_{12} = 0\}\,, \tag{4.16}$$

[7]This solution is unique if one imposes boundary conditions such that calibrated surface is the asymptotically the same as the original intersection.

which corresponds to

$$L \simeq \Sigma \times \mathbb{R} \cup \mathbb{R}^3\,. \tag{4.17}$$

It is instructive to compare this result with the analysis of Gukov and Tong [9], who use D6 brane intersections in order to construct manifolds of G_2 holonomy in M-theory. For a configuration of three D6 branes intersecting at angles of $2\pi/3$ they find a very similar structure to (4.17). Here we obtain this structure from F- and D-flatness in the world-volume gauge theory.

The Higgs branch of the triple intersection provides a controlled setting to illustrate the Higgs mechanism which is expected to occur in intersecting brane constructions containing the Standard Model [10, 11, 12]. Typically, there one has (besides the brane that yields the hyper-charge) three stacks of branes whose gauge groups one identifies with the left handed SU(2) and right handed U(1) and the color SU(3). The fields at the intersections of the two electro-weak stacks are the Higgs fields whereas the quarks live at the other two intersections as they are in the bifundamental of the color and the electroweak symmetry group. In this context the Higgs is tachyonic. Unlike the supersymmetric brane recombination we have described, a precise treatment of brane recombination in the intersecting brane-world models is lacking and would seem to require string field theory, although some interesting effective field theory descriptions have been proposed [18, 20, 19, 47]. Upon condensation of the Higgs field, the two "electro-weak" branes resolve into one giving a diagonal subgroup after chiral symmetry breaking. The stacks corresponding to the left- and right-handed weak gauge groups do not intersect anymore with the stack of the color gauge group (as we found above: we can only give one B a vev as this renders the others massive), making the quarks that stretch between the color and the weak stacks massive.

5. Scattering between recombined branes

We now change gears somewhat and, returning to the case of a D3-brane on the holomorphic curve $wy = c$, compute the transmission amplitude for a particle at $x \to \infty$ to reach $y \to \infty$.

This is similar to a calculation performed in [48] for the BIon [40]. We will find that the transmission approaches 1 for $c \to \infty$ and vanishes for $c \to 0$. This result is consistent with the expectation that the D3-brane on the holomorphic curve becomes two separate orthogonal branes at the point $c = 0$.

To compute the transmission amplitude we will study the two-point function or propagator of light fields living on the resolved brane intersection. Consider the massless wave equation with respect of the induced metric on the brane. The 2-point function will obey this covariant wave equation:

$$0 = \Box_g \psi \tag{5.1}$$

where $\Box_g$ is the d'Alambertian corresponding to the metric that is pulled back to the curve $wy = c$. After a rotation in the w and y planes we can assume that c is real and non-negative. We write $y = u + iv$ and $w = r\, e^{i\phi}$ and find

$$ds^2 = dr^2 + r^2 d\phi^2 + du^2 + dv^2$$

$$= \left(1 + \frac{c^2}{r^4}\right)(dr^2 + r^2 d\phi^2)\,. \tag{5.2}$$

We see that this metric has the expected asymptotics: For large r the prefactor trivializes and we have a flat two-dimensional metric. For $r \to 0$, the prefactor becomes c^2/r^4 and after a change of variables $u = \sqrt{c}/r$ again we have a flat metric.

The two-point function from $r = \infty$ to $r = 0$ tells us how much of a wave that comes from infinity on the horizontal branch of the brane is transmitted to the vertical branch of the brane. This problem is like a quantum mechanical scattering problem, associated with which is a concerved current

$$j = \bar{\psi} d\psi - \psi d\bar{\psi}\,, \qquad d * j = 0\,. \tag{5.3}$$

Unfortunately, we do not have an analytic solution to the wave equation in the above metric. So we have to resort to numerical methods. However, in the asymptotic regions $r \to \infty$ and $r \to 0$ the radial wave equation in flat space can be solved in terms of Hankel functions $H_\pm$. For concreteness we consider the S-wave for which ψ is only a function of r. The case of non-trivial ϕ-dependence of can be treated similarly.

At large r, every solution to the wave equation has the form

$$\psi(r) = AH_+(r) + BH_-(r)\,, \qquad H_\pm(r) \to \sqrt{\frac{1}{r}} e^{\pm ir}\,. \tag{5.4}$$

A is the coefficient for the outgoing wave, B correspondingly for the ingoing one. Accordingly the current is

$$j_r = 2riH_+(r)H_-(r)(|A|^2 - |B|^2)dr\,. \tag{5.5}$$

For $r \to 0$ we have similarly

$$\psi(r) = ah_+(r) + bh_-(r)\,, \qquad h_\pm(r) = H_\pm\left(\frac{c}{r}\right) \tag{5.6}$$

and

$$j = -\frac{ic}{r} h_+(r)h_-(r)(|a|^2 - |b|^2)dr\,. \tag{5.7}$$

Now we start with initial values for large r that correspond to $B = 0$ and integrate the full equation numerically using *mathematica*. At small r we fit this numeric solution to the form (5.6) and read of the coefficients a and b. By equating the two asymptotic expressions for the current j_r, which satisfies $\partial_r j_r = 0$ we find

$$1 = \frac{r^2 H_+(r)H_-(r)}{ch_+(r)h_-(r)} \frac{|A|^2}{|a|^2} + \frac{|b|^2}{|a|^2}\,. \tag{5.8}$$

The first term is the coefficient of transmission, the second one the coefficient of reflection. Figure 3 shows the transmission plotted against the value of $\tilde{b}b$:

For small values of c the transmission vanishes. That is, in the limit of two branes intersecting on a line the degrees of freedom on the two branes decouple. On the other hand, for large c, when the intersection is deformed away, the transmission approaches 1 and there is really only one brane left and the degrees of freedom in the horizontal and vertical asymptotic regions are connected as one would expect it for a single brane.

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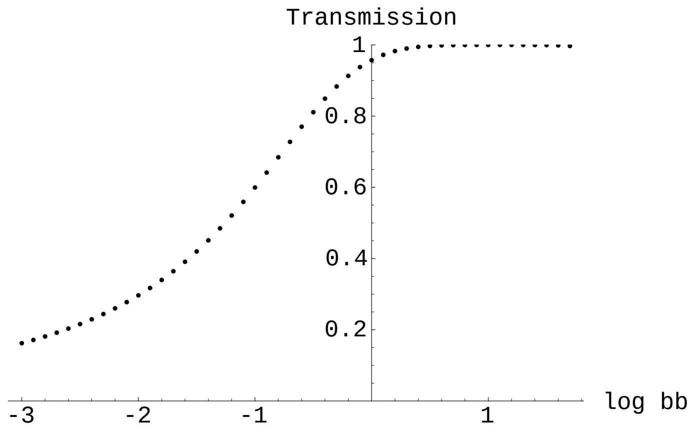

Figure 3: Transmission for different values of the deformation $\log c$.

6. Conclusions

In this note we have given a few examples in which calibration equations arise as solutions of F and D-flatness conditions when the action for a brane is written in terms of a lower dimensional superspace. In fact this result is quite general and should include various calibration conditions which we have not mentioned here.

The lower dimensional superspace formalism is particularly useful to write the action for intersecting branes, as it facilitates writing the couplings between ambient fields and the twist fields localized at the intersections. We have explicitly shown how intersecting branes recombine into smooth calibrated manifolds when the twist fields condense on the Higgs branch.

In the case of special lagrangian manifolds, the Yang Mills description suffices to give the lagrangian condition, arising from the F-term in the appropriate lower dimensional superspace. A linearized special-lagrangian condition arises when D-flatness is also considered. The non-linear terms vanish for the special case of a holomorphic curve times a line, which we have shown to arise on the Higgs branch of the gauge theory describing a triple intersection of D-branes. It would be interesting to obtain the full non-linear special lagrangian condition from a D-flatness condition in supersymmetric version of the Dirac-Born-Infeld theory.

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